

# Prediction of Error due to Eccentricity of Hole in Hole-Drilling Method Using Neural Network

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The measurement of residual stresses by the hole-drilling method has been used to evaluate residual stresses in structural members. In this method, eccentricity can usually occur between the hole center and rosette gage center. In this study, we obtained the magnitude of the error due to eccentricity of a hole through the finite element analysis. To predict the magnitude of the error due to eccentricity of a hole in the biaxial residual stress field, it could be learned through the backpropagation neural network. The prediction results of the error using the trained neural network showed good agreement with FE analyzed results.

**Key Words :** Hole-Drilling Method, Finite Element Analysis, Residual Stress, Neural Network

## 1. Introduction

Residual stress means the stress that exists inside the machinery or structures without external loads. Such residual stress is superposed onto the service stress and affects the fatigue life of machinery or structures significantly. Especially, for structures connected with a material part that experienced the hot forming procedure or welding, residual stress may increase to the level of yield stress. For such a reason, if the structures were designed on the assumption of no residual stress, safety problem may be significant. As a result, the magnitude of residual stress should be accurately considered in the design of structures, and accurate measurement of residual stress is very important.

The hole-drilling method (HDM) makes a small hole on the metal surface that has residual

stress and measures the relieved stress with a strain gage. It is widely used in measuring the residual stress on the surfaces. Mathar (1934) first suggested this method in the 1930s and constant developments were achieved in its theory and applications. Further studies on more accurate and simpler measurement instruments and methods have been accomplished (Flaman et al., 1986; Tootoonian et al., 1995; Schajer et al., 1997).

The current standard examination method of the hole drilling method is specified in ASTM E837-99. Because the measurement results vary by the locations of the holes, vertical hole drilling is assumed in ASTM and the eccentricity between the center of the strain gage and the center of the hole is restricted to be under 0.025 mm. However, in many cases, measurement objects are often irregular, and even when special measurement tools such as RS-200 are used, accurate hole drilling is quite difficult to perform. As a result, when holes are drilled with an eccentricity, errors are generated due to the eccentricity of the hole. When hole drilling is carried out, it is required that some error should be anticipated in advance.

Various studies have been performed on the

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effects of eccentricity of the drilled hole in the hole-drilling method. Ajovalasit (1979) performed a theoretical study and Wang (1990) proposed a method that corrects the relieved strain coefficient using numerical analysis. Kim et al. (2002) proposed prediction of the error due to the eccentricity of hole and correction of the error in the uniaxial residual stress field using finite element and numerical analyses. However, theoretical studies until now have set the equations based on Kirsch's solution (1970) that is intended for the through-hole cases. When the blind hole is drilled, accurate theoretical solutions for the effects of the eccentricity have not been proposed due to many difficulties such as setting boundary conditions, etc. Moreover, most studies on the finite element and/or numerical analysis have been performed by simplifying the stress field to cases of the uniaxial residual stress field.

When the hole-drilling method is applied to the general biaxial residual stress field, the eccentricity of the hole includes error for the measured value. The magnitude of the error depends on multiple variables including the stress ratio, the magnitude and direction of eccentricity and so on. Various algorithms have been developed to predict the results of these multiple variables, and studies to predict the results using artificial neural networks have been performed actively (Cho and Joo, 2000 ; Inamdar et al., 2000). When accurate and sufficient learning data are available, artificial neural networks can be used to effectively predict the result of any input variable.

In this study, we established a biaxial residual stress field model and obtained the magnitude of the error due to eccentricity of hole through the finite element analysis. In order to predict the magnitude of the error due to eccentricity of hole in the biaxial residual stress field, it could be learned through the backpropagation neural network.

## 2. Training Algorithm of Neural Network

The artificial neural network (Fausett, 1994) is a model that has been made by imitating the brain

structure and learning habits of humans. It generalizes data through neural network training similar to the learning methods of the human brain. Fig. 1 shows the generalization process of the artificial neural network. For generalization, training examples of the question must be proposed with an input pattern to the neural network. The "education" process involves repeated input patterns to the neural network until they are "learned". When learning is completed successfully, examples are used to test the neural network and the trained network proposes appropriate prediction results.

The backpropagation neural network is one of the most widely used learning models proposed by Rumelhart and others (1986). This model consists of an input layer a hidden layer, and an output layer. While several hidden layers can be used in this model, it is known that only one hidden layer is sufficient. Each layer is composed of many units and the units between each layer are connected with weights. The learning algorithm of the back propagation neural network is performed in two steps. In the first step, input is proposed to the network, which creates an output by being propagated in all directions of the network. The output unit error is calculated by multiplying a differential coefficient by the difference between the output and target value. In the second step, the error signal is propagated backwards through the network and the weight is corrected based upon it. The control regulations of the weight is expressed as :

$$w_{jk}(new) = w_{jk}(old) + \alpha \delta_k z_j + \mu \Delta w_{jk}(old) \quad (1)$$

where  $\delta_k = (T_k - O_k) f'(O_{in_k})$

It Eq. (1)  $w_{jk}$  is the weight from  $j$ th unit to the  $k$  th unit and  $\Delta w_{jk}$  is the amount of change in the weight.  $z_j$  is the unit output of the  $j$ th-hidden layer,  $T_k$  is the target value, and  $O_k$  is the output value of the  $k$ th unit.  $O_{in_k}$  is the input value of the unit in the  $k$ th output layer, and  $f'(O_{in_k})$  is the differential value of the activation function at the  $k$ th unit. A sigmoid function has been used in this work as the activation function.

$$f(x) = \frac{1}{1 + e^{-x}} \quad (2)$$

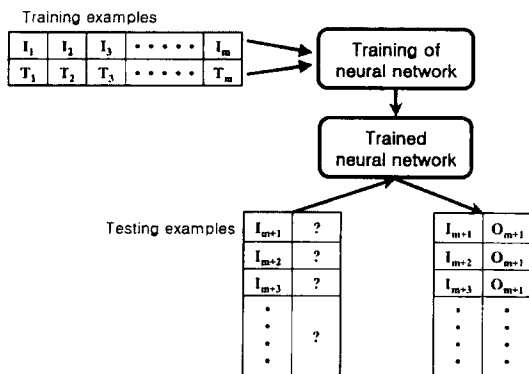


Fig. 1 Generalization procedure of neural network

The learning rate controls the learning speed of the neural network and the momentum parameter increases the learning speed and prevents the neural network learning from reaching the local minimum. Difference between the target and output values of each unit is defined as :

$$E(w) = \frac{1}{2} \sum_{k=1}^m (T_k - O_k)^2 \quad (3)$$

The learning of the neural network is to control the weight by minimizing the learning error of Eq. (3) with the steepest gradient descent method. When the output value approaches the target value, learning terminates with the learning error smaller than the defined value.

### 3. Finite Elements Analysis

#### 3.1 Analysis model and method

In this study, we set the analysis model illustrated in Fig. 2. The hole drilling strain gage was attached to the center of the plate with biaxial

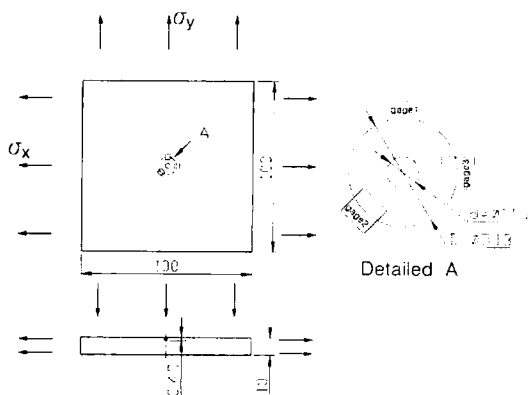


Fig. 2 Model configuration

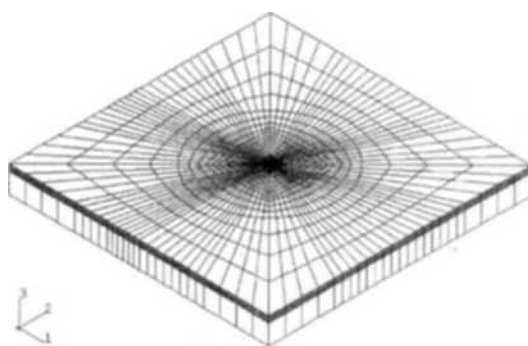


Fig. 3 3-Dimensional finite element model

stress. The strain gage of the TEA-XX-062RK-120 (Measurements Group) was used and the hole diameter  $d$  was 1.57 mm and the diameter of strain gage  $D$  was 5.13 mm. The finite element mesh was three-dimensional to correspond to the strain gage model and was divided into 7 layers to remove the elements of the hole part while drilling the hole. As to the material properties, the Young's modulus of 205 GPa and Poisson's ratio

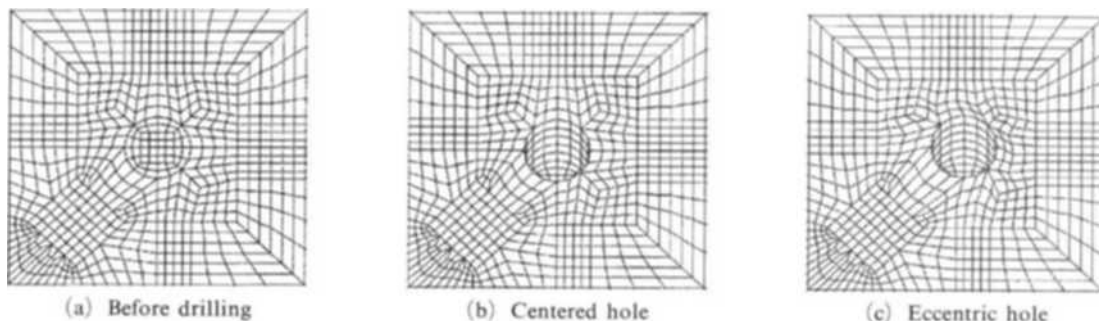


Fig. 4 Strain gage and hole part

of 0.3 were used. Fig. 3 shows the finite element mesh used in this analysis, and Fig. 4 shows the details of the strain gage before and after drilling with a centered hole, and drilling with an eccentricity of the hole. Particularly, one strain gage is composed of 48 elements. The average strains of 48 elements were calculated and they are the strains at each position of the strain gage. The 14,210 of 8 nodal solid elements were used as the model and the number of nodes was 16, 648. The program used in this finite element study is ABAQUS.

We analyzed the error due to eccentricity of hole to determine the neural network learning data. The stress ratio  $\sigma_y/\sigma_x$  has been divided into 10 levels ranging from  $-5$  to  $5$ , eccentricity  $e$  has been divided into 4 levels ranging from  $0.025$  mm that is the ASTM eccentric allowance to  $0.375$  mm, which is about 0.5 times the magnitude of hole radius  $r$ . As for eccentricity direction, it was divided into 12 levels starting from the No. 1 strain gauge and moving  $30^\circ$  clockwise. We performed analysis to acquire a total of 480 neural network training patterns.

**3.2 Verification of analysis model**

First, we conducted the analysis of residual stress when the hole was not inclined to verify the feasibility of the finite element mesh. We calculated the average strain of each elements at each strain gage before and after drilling the hole. With the calculated strains, the maximum stresses and the direction are predicted as :

$$\sigma_{1, 2} = \frac{\epsilon_3 + \epsilon_1}{4A} \pm \frac{\sqrt{(\epsilon_3 - \epsilon_1)^2 + (\epsilon_3 + \epsilon_1 - 2\epsilon_2)^2}}{4B} \quad (4)$$

$$\beta = \frac{1}{2} \tan^{-1} \left( \frac{\epsilon_3 + \epsilon_1 - 2\epsilon_2}{\epsilon_3 - \epsilon_1} \right)$$

**Table 1** Verification results for finite element model

Applied residual stress (MPa)		HDM analysis result			Error of stress (%)
$\sigma_x$	$\sigma_y$	$\sigma_1$ (MPa)	$\sigma_2$ (MPa)	$\beta$ (deg)	
100	0	99.95	0.0	0.07	0.05
100	100	99.43	99.43	0.01	0.57
100	-100	100.09	-100.04	0.01	0.09

In Eq. (4), the calibration coefficients  $\bar{A}$ , and  $\bar{B}$  are obtained using finite element analyses with ASTM E 837. The calibration coefficients  $\bar{A}$  and  $\bar{B}$  were obtained such that  $\bar{A}$  is  $-3.0225 \times 10^{-7}$ ,  $\bar{B}$  is  $-6.5565 \times 10^{-7}$ . As a result of the analyses for the three different distributions of residual stresses, the error was within 1% when the hole was not eccentric, and the Table 1 shows the results.

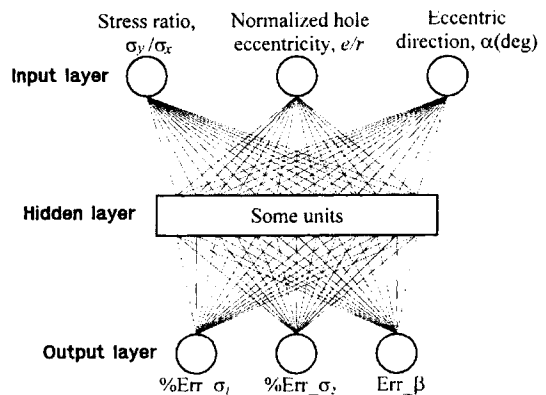
**4. Neural Network Training**

The neural network model for the prediction of error due to eccentricity is illustrated in Fig. 5. The input layer used 3 units of stress ratio  $\sigma_y/\sigma_x$ , normalized eccentricity  $e/r$ , and eccentric direction  $\alpha$ . The output layer used the % error of principal stress 1 and 2 which is caused by eccentricity of hole and calculated as shown in Eq. (5) and direction error of principal stress  $Err_\beta$ .

$$\%Err_{\sigma_i} = \frac{\sigma'_i - \sigma_i}{\sigma_i} \times 100 \quad (5)$$

$\%Err_{\sigma_i}$ , in Eq. (5) is % stress error ( $i=1, 2$ ),  $\sigma'_i$  is the principal stress when a hole is eccentric, and  $\sigma_i$  is the principal stress for centered drilling. Neural network learning using the model of Fig. 5 was performed in two separate cases where the stress ratio was positive and negative.

We used 240 patterns of neural network learning when the stress ratio was negative and Fig. 6



**Fig. 5** Architecture of neural network for error prediction due to eccentricity of hole in HDM

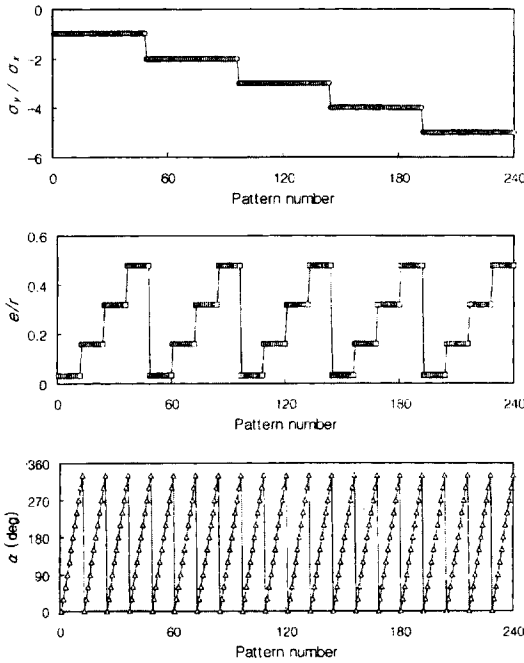


Fig. 6 Input values for negative stress ratio

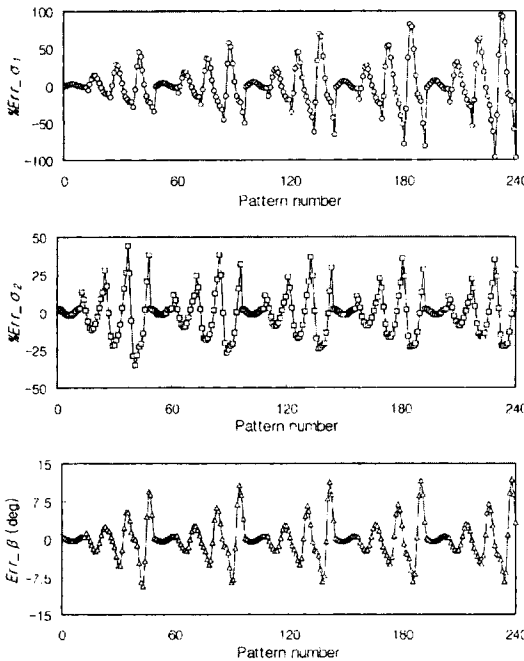


Fig. 7 Target values for negative stress ratio

shows the input value of the learning data. Fig. 7 shows  $\%Err_{\sigma_1}$ ,  $\%Err_{\sigma_2}$ , and  $Err_{\beta}$ , which are target values when the stress ratio is negative.

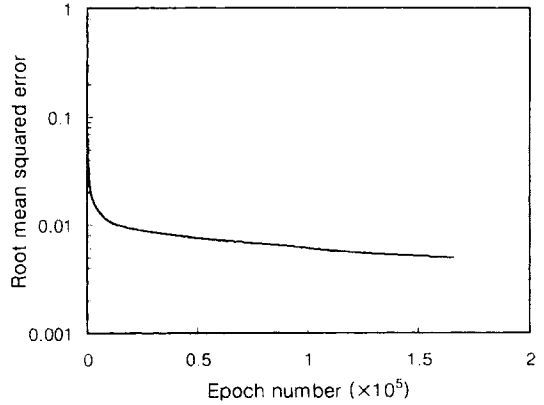


Fig. 8 Root mean squared error versus epoch number for negative stress ratio

In this study, we performed neural network learning using a backpropagation neural network program with the hidden layer units of 80, the learning rate of 0.1, and the momentum parameter of 0.8. Learning was programmed to terminate when the root mean squared error (RMSE) reached 0.005 or epoch number repeated for 300,000 times.

$$RMSE = \sqrt{\frac{\sum_{k=1}^m (T_k - O_k)^2}{N}} \quad (6)$$

Fig. 8 shows the root mean squared error according to the epoch number. The root mean squared error reached the target error of 0.005 after 166,152 times of repeated learning. Success of neural network learning is judged by comparing the output and input values of each unit from the neural network output layer. Fig. 9 shows the neural network output and input values together and shows that all three of the output values match the target values, which implies that neural network learning has been achieved successfully.

When the stress ratio was positive, neural network learning is performed in the same way as the negative stress ratio. We used 240 input patterns, and learning data input values and target values are shown in Fig. 10 and Fig. 11. The momentum parameter of 0.8, the learning rate of 0.1, and the 80 hidden layer units were used as neural network learning conditions and the final

neural network learning was performed until the

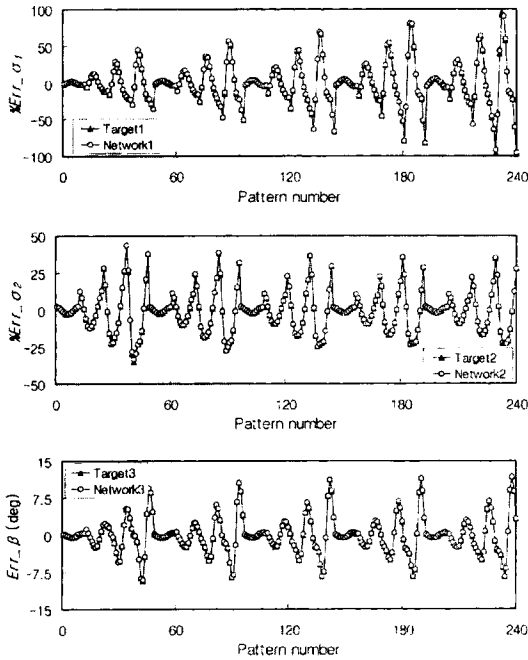


Fig. 9 Comparison network output values with target ones for negative stress ratio

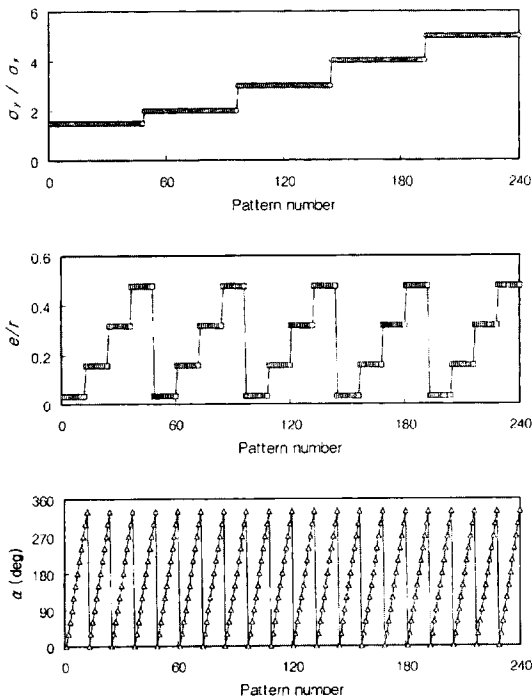


Fig. 10 Input values for positive stress ratio

root mean squared error reached 0.005. The root mean squared error reached the target error after 207,558 repeated learning trials. The root mean squared error according to epoch number is shown in Fig. 12. Fig. 13 shows the neural network output and target values after learning when the stress ratio was positive and it shows that all three of the output values match the target values.

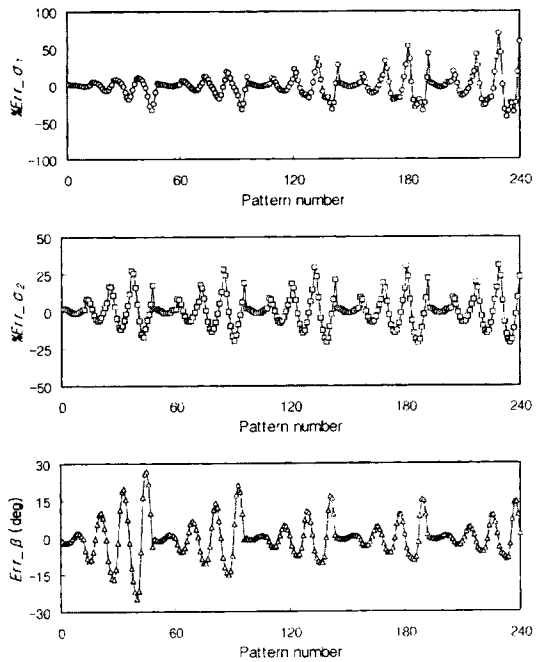


Fig. 11 Target values for positive stress ratio

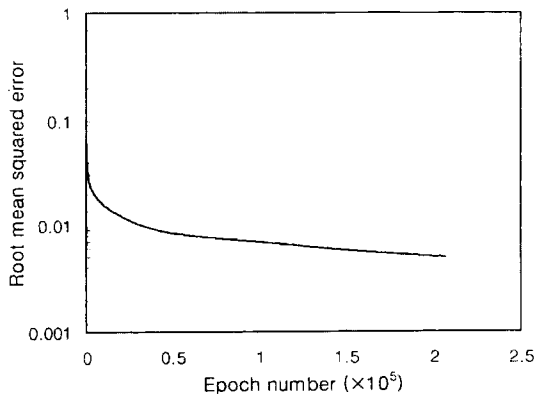


Fig. 12 Root mean squared error versus epoch number for positive stress ratio

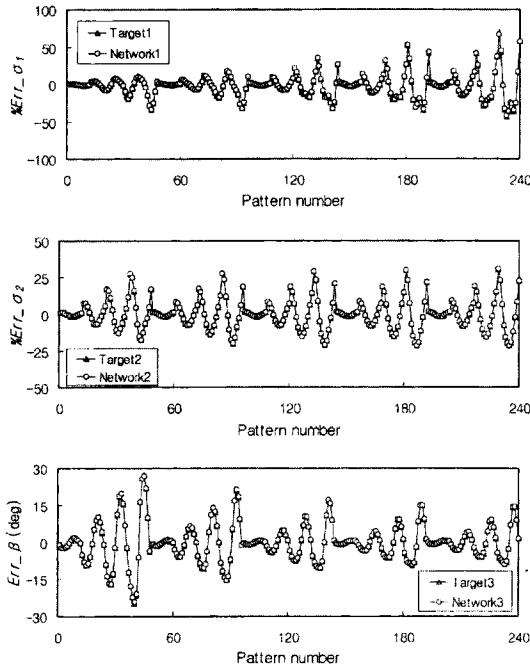


Fig. 13 Comparison network output values with target ones for positive stress ratio

5. Prediction of Eccentric Error

When any input value is tested using the trained neural network after successful learning, the network provides an appropriate prediction result in accordance with the input. We presented input values for voluntary stress ratio, magnitude and direction of eccentricity to the trained neural network and predicted the error magnitude for the input.

Table 2 and Table 3 show a comparison of the error results that were acquired by the finite element analysis and predicted by the neural network. Table 2 shows the results when the stress ratio was negative and Table 3 shows the results when the stress ratio was positive. We compared the predicted results of neural network and finite element analysis results for two voluntary input values. Although the data on one relative error exceeded the engineering error range, it was a sufficiently small value from an absolute error standpoint. Therefore, the neural network prediction values generally matched the results of finite element analysis.

Table 2 Predicted results for negative stress ratio (a) Case 1

Input value		Output value (error due to eccentricity)		
			FE analysis	Neural network
Stress ratio, $\sigma_y/\sigma_x$	-5.83	%Err $_{\sigma_1}$	31.76	31.27
Normalized eccentricity, $e/r$	0.191	%Err $_{\sigma_2}$	-1.42	-1.61
Eccentric direction, $\alpha$ (deg)	80	Err $_{\beta}$ (deg)	-2.34	-2.26

(b) Case 2

Input value		Output value (error due to eccentricity)		
			FE analysis	Neural network
Stress ratio, $\sigma_y/\sigma_x$	-1.4	%Err $_{\sigma_1}$	11.89	11.63
Normalized eccentricity, $e/r$	0.2541	%Err $_{\sigma_2}$	-16.23	-16.44
Eccentric direction, $\alpha$ (deg)	160	Err $_{\beta}$ (deg)	-4.04	-4.17

Table 3 Predicted results for positive stress ratio (a) Case 1

Input value		Output value (error due to eccentricity)		
			FE analysis	Neural network
Stress ratio, $\sigma_y/\sigma_x$	1.5	%Err $_{\sigma_1}$	7.86	7.57
Normalized eccentricity, $e/r$	0.3185	%Err $_{\sigma_2}$	13.97	14.39
Eccentric direction, $\alpha$ (deg)	45	Err $_{\beta}$ (deg)	-15.4	-14.7

(b) Case 2

Input value		Output value (error due to eccentricity)		
			FE analysis	Neural network
Stress ratio, $\sigma_y/\sigma_x$	6.0	%Err $_{\sigma_1}$	-38.63	-36.98
Normalized eccentricity, $e/r$	0.3312	%Err $_{\sigma_2}$	-13.21	-12.60
Eccentric direction, $\alpha$ (deg)	135	Err $_{\beta}$ (deg)	-5.66	-5.90

In case of the same type of strain gage, we believe that if the trained neural network described in this research is used, we can effectively predict the error magnitude due to eccentricity of hole without performing additional finite element analysis processes.

## 6. Conclusions

In order to predict the measured error due to eccentricity of hole when residual stress was measured by the hole-drilling method, we found the error according to stress ratio and found the magnitude and direction of eccentricity using finite element analysis. We then generalized the results using artificial neural networks.

We decided the neural network learning conditions and performed final neural network learning individually when the stress ratio was negative and positive. The output values, after neural network learning, showed good agreements with the target values. The results of error predictions according to voluntary stress field, the magnitude and direction of eccentricity using the trained neural network generally matched the results of the finite element analysis. This means that the eccentricity error in the hole-drilling method can be effectively predicted using the results of this study.

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